A Study of Dielectric Resonators Based on Two-Dimensional Fast Wavelet Algorithm

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***Abstract*—This letter reports the implementation of orthonormal wavelets for the moment method characterization of three-dimensional dielectric structures. The formulation is based on a 3-D volume integral equation, which is solved numerically using a 2-D multiresolution analysis in conjunction with a sub-domain pulse basis. The use of multiresolution expansions leads to highly sparse linear systems that can be solved very efficiently using the bi-conjugate gradient method. To speed up the numerical evaluation of moment integrals, the fast wavelet algorithm (FWA) has been employed.**

I. Introduction

The numerical modeling of dielectric resonators has been investigated by several authors [1]–[3]. As new complex dielectric structures find their ways into microwave circuit designs, rigorous numerical techniques with high levels of accuracy and computational efficiency are needed that can go beyond the inherent limitations of approximate methods. It has been shown in the past two years that the use of multiresolution expansions in the moment method solution of electromagnetic problems leads to the generation of highly sparse linear systems [4]–[6]. This approach eliminates the traditional bottleneck of the integral-based formulations due to the fullness of moment matrices. The results reported to date mostly deal with 2-D or planar structures using one-dimensional multiresolution expansions. In such problems the number of unknowns, and therefore the size of the linear systems, are quite reasonable and the sparsity of the moment matrices, although favorable, is not a critical factor. However, in 3-D problems such as dielectric resonators that involve the numerical solution of volume integral equations, the storage and inversion of very big, densely populated matrices can easily exceed the capability of available computing resources. In such cases, it is vital to take full advantage of any matrix sparsity if possible.

In this letter we present a 3-D space-domain wavelet-based moment method formulation of dielectric resonators. To fully exploit the resulting sparsity of moment matrices, the bi-conjugate gradient (BiCG) method is used for the numerical solution of the sparse linear systems. To this end, an incident field is retained in the formulation of the integral equation to excite the dielectric structure at a certain frequency. This approach has the advantage of providing the field distribution and resonant frequency simultaneously. Moreover, by changing the polarization of the incident field, one can easily study various resonant modes of the structure. To reduce the computational load due to the numerical evaluation of moment integrals, we have employed a very interesting property of the multiresolution analysis. This property is the fast wavelet algorithm (FWA), which enables one to compute the multiresolution expansion coefficients at each resolution level from a knowledge of the coefficients at one higher level.

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II. Formulation and Numerical Implementation

Consider a rectangular dielectric resonator of dimensions $a \times b \times h$ with a relative permittivity of $\epsilon_{rg}(\boldsymbol{r})$, where $\boldsymbol{r}$ is the position vector. The background geometry can be any planar layered substrate configuration, and the z axis is assumed to be normal to the plane of substrate layers. The finite dielectric region is modeled by an equivalent volume polarization current that is defined to be proportional to the total electric field inside the dielectric resonator and vanishes outside this volume. This current acts as a radiation source embedded in the background geometry. Then, the total electric field is expressed in the following way:

$$\boldsymbol{E}(\boldsymbol{r}) = -jk_0 Z_0 \int\int\int_V \overline{\overline{\boldsymbol{G}}}_e(\boldsymbol{r} \mid \boldsymbol{r}') \cdot \boldsymbol{J_p}(\boldsymbol{r}') dv' + \boldsymbol{E^i}(\boldsymbol{r}) \tag{1}$$

where k_0 and $Y_0 = 1/Z_0$ are the free-space propagation constant and characteristic admittance, respectively, $\boldsymbol{E^i}(\boldsymbol{r})$ is the incident electric field, and $\overline{\overline{\boldsymbol{G}}}_e(\boldsymbol{r} \mid \boldsymbol{r}')$ is the dyadic Green's function of the substrate structure. Thus, a volume integral equation can be derived for the unknown polarization current.

To obtain a high degree of sparsity in the moment method implementation, we expand the volume current $\boldsymbol{J_p}$ in a 2-D multiresolution basis in the x-y plane and a sub-domain pulse basis along the z axis in the following manner:

$$\boldsymbol{J_p}(\boldsymbol{r}) = \sum_j \sum_q \boldsymbol{a_{jq}} F_j(\frac{x}{d_0}, \frac{y}{d_0}) p_q(z) \tag{2}$$

where $F_j(x, y)$ is a 2-D multiresolution basis function, d_0 is the characteristic length of the structure, and $p_q(z)$ is a sub-domain pulse basis function. The construction of 2-D multiresolution expansions has been discussed in detail in [7]. Such expansions are generated by dilating and shifting a 2-D scaling function and three types of horizontal, vertical, and diagonal 2-D wavelets. By testing the discretized integral equation using Galerkin's technique, a linear system of matrix equations is obtained. This system is solved iteratively using a pre-conditioned bi-conjugate gradient (BiCG) method. The resulting moment matrix is highly sparse, and by performing a thresholding procedure it can easily be stored using sparse storage techniques. In addition, due to the vanishing moments property of wavelet functions, many negligible moment interactions can be predicted in advance and their computation is spared [8].

In order to speed up the numerical integrations, the fast wavelet algorithm is implemented. The one- and multidimensional versions of this algorithm have been described in [9]. The fast wavelet algorithm establishes the following relations between the multiresolution expansion coefficients of an arbitrary function $f(x)$:

$$< f, \phi_{m,n} > = \sum_{k \in Z} h_{k-2n} < f, \phi_{m+1,k} >,$$
$$< f, \psi_{m,n} > = \sum_{k \in Z} g_{k-2n} < f, \phi_{m+1,k} > \qquad (3)$$

where $\phi_{m,n}(x)$ and $\psi_{m,n}(x)$ are the dilated and shifted scaling function and wavelet with resolution and shift indices m and n, respectively, and $\{h_n\}$ and $\{g_n\}$ are discrete sequences characteristic to the multiresolution analysis. The digital filters represented by (3) involve a discrete convolution plus a decimation by two. This equation implies that the scaling and wavelet coefficients at each resolution level m can be computed recursively from the scaling coefficients at resolution level $m+1$. Thus, using the fast wavelet algorithm we can confine the time-consuming task of numerical integration to the 2-D scaling functions at the highest resolution of the problem. All the moment integrals at different resolution levels including inter-level and intra-level interactions can then be computed recursively by simple discrete convolutions.

III. Numerical Results

To validate our formulation, a rectangular dielectric resonator of dimensions 10 mm × 8 mm × 5 mm with a relative permittivity of $\epsilon_{rg} = 20$ immersed in the free space has been considered. The resonator is excited by a normally incident plane wave with a fixed linear polarization. By changing the polarization of the incident field, various resonant modes of the structure are excited. For the geometry under study, the dominant mode is a TM^x_{111} mode assuming that the largest dimension of the resonator is aligned along the x axis. By varying the excitation frequency, one can determine the resonant frequency of each mode, where the stored electric energy inside the resonator reaches a maximum value.

For the expansion of the polarization current along the z direction, 1–4 pulses usually provide very satisfactory results. The Battle-Lemarie multiresolution analysis has been used to construct the 2-D wavelet expansion in the x-y plane [8]. The characteristic length d_0 of problem is taken to be the dielectric wavelength $\lambda_g = \lambda_0/\sqrt{\epsilon_{rg}}$. The initial resolution level of $m_0 = 2$ is chosen to obtain an initial crude approximation using 2-D scaling functions, and then the 2-D wavelets at two resolution levels $m = 2, 3$ are used to further improve the approximation. The resulting moment matrix, as expected, is

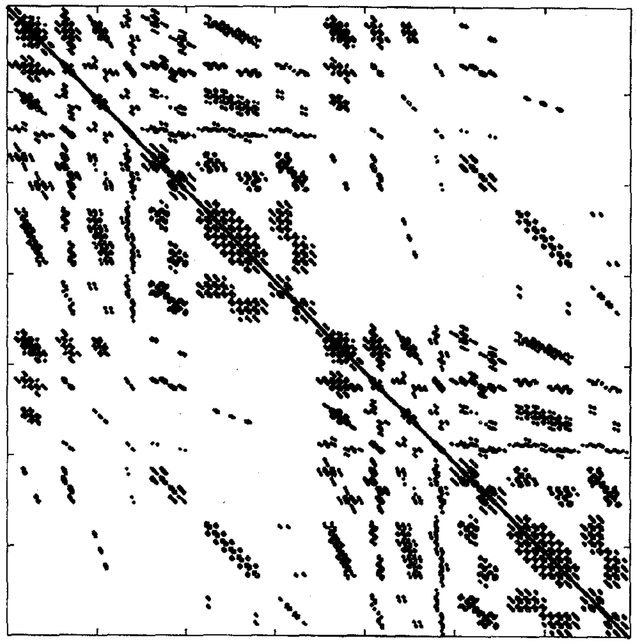

Fig. 1. The structure of moment matrix of a dielectric resonator after applying a threshold of 1%.

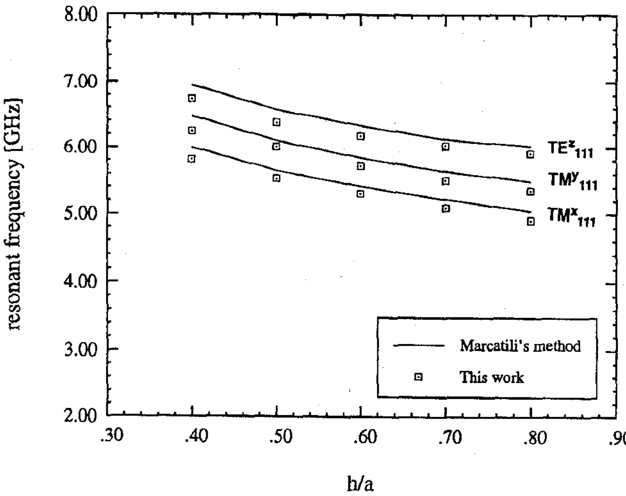

Fig. 2. Variation of resonant frequencies of the dominant modes of a dielectric resonator as a function of the aspect ratio h/a.

highly sparse in the sense that a large number of its entries are very small in magnitude when compared to the largest entry. Fig. 1 shows the structure of the moment matrix after applying a threshold of 1%. In this case, the expansion basis consists of 2 pulse functions and a total of 215 2-D multiresolution expansion functions, and the sparsity of the moment matrix is 99.22%. Fig. 2 shows the variation of the resonant frequencies of the first three modes of the dielectric resonator considered above as a function of the aspect ratio h/a. The results have been compared to those based on Marcatili's approximation [1] and a good agreement is observed.

To better envision the computational savings, we have compared our results with the conventional method of moments using 3-D pulse expansions. The moment matrix of Fig. 1

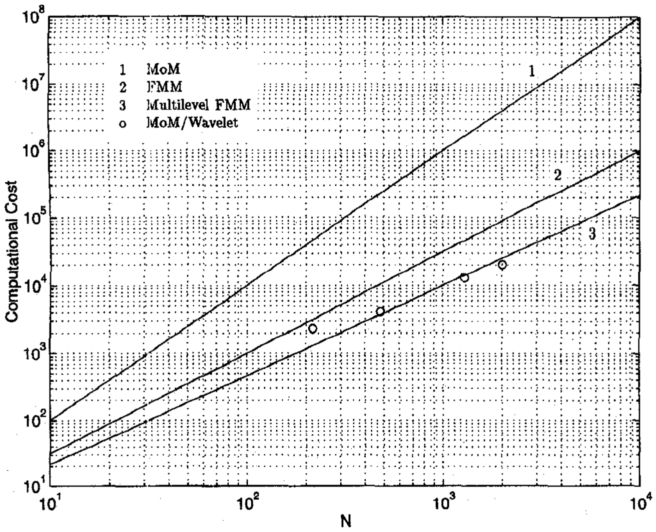

Fig. 3. Comparison of computational cost among various integral-based techniques.

after thresholding contains 12 980 nonzero elements. These elements can be arranged into a 1-D array and easily stored using a sparse storage scheme. A conventional MoM formulation using 3-D pulse expansions would typically require an $8 \times 6 \times 4$ discretization mesh for the same geometry, which amounts to 576 unknowns. Since the resulting moment matrix is fully populated, all of the 576^2 elements must be stored for the inversion process. Thus, the wavelet approach offers a reduction of more than 96% in the effective size of the numerical problem. It is also well known that in the conventional method of moments using an iterative linear system solver, the most expensive part of the computation is the vector-matrix multiplication, which involves a computational cost of $O(N^2)$, N being the number of unknowns. This cost has been reduced to $O(N^{3/2})$ through the development of the fast multipole method (FMM) and has further been improved to $O(N^{4/3})$ using multilevel algorithms [10]. The wavelet-based approach of this letter involves a computational cost of order of $[1 - \alpha(N)]N^2$, where $\alpha(N)$ is the sparsity of the moment matrix. Fig. 3 compares the computational costs of various techniques as a function of the size of the problem.

IV. Conclusion

A very efficient sparse moment method formulation has been presented for the analysis of 3-D planar dielectric structures, which is based on the concepts of orthonormal wavelet theory. The combination of sparse matrix techniques and the fast wavelet algorithm make this approach an effective and fast tool for the study of 3-D electromagnetic problems.

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